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Sustainable Inventory Model with Trade Credit Policy and Carbon Emission Cost under Exponential Lead Time Demand

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ABSTRACT
Sustainable inventory management has become a key focus area for modern supply chains as organizations face increasing pressure to balance economic performance with environmental responsibility. One major challenge in this direction is the reduction of carbon emissions, which arise from activities such as production, transportation and warehousing. At the same time, trade credit has become an important financial mechanism through which suppliers allow buyers to delay payment for a specified credit period. Another critical factor affecting inventory cost is uncertainty in lead time demand. Variability in order processing, transportation delays and market fluctuations make lead time unpredictable. Modelling lead-time demand using an exponential probability distribution provides a realistic approach for minimizing stock out risk and determining an appropriate reorder level. This research develops a sustainable inventory model by jointly integrating carbon emission cost, trade credit policy and exponential lead time demand. The objective is to determine the optimal order quantity, reorder level their corresponding total cost and to analyse how changing parameters influences the overall system, thereby suggesting good managerial practices that lowers the total cost while balancing financial and environmental considerations.

KEYWORDS
Sustainable Inventory model; lead time demand; exponential distribution; trade credit; carbon emissions

1. Introduction

Inventory management plays a critical role in the efficient functioning of supply chains and business operations. Traditionally, inventory models were developed with the primary objective of minimizing total cost or maximizing profit by determining optimal order quantities, lead time and replenishment policies. However, the business environment has changed significantly in recent decades due to rising awareness about environmental responsibility and sustainability. Global concerns over climate change, carbon footprint reduction and strict environmental regulations have forced industries to revise their traditional inventory strategies. As a result, there is a growing emphasis on developing sustainable inventory models that integrate both economic and environ-

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mental considerations. Carbon emissions play a pivotal role in sustainable inventory control because every activity in a supply chain contributes to environmental pollution. Ignoring these emissions may lower short-term operational costs but lead to higher long-term environmental damage and regulatory penalties. Integrating carbon emission cost into inventory decision-making not only helps organizations meet environmental regulations such as emissions quota, carbon tax and cap-and-trade systems but also strengthens their corporate social responsibility, reputation and global competitiveness. Sepehri et al. (2021) have stated that including controllable carbon emission cost in inventory models is essential for organizations aiming to improve sustainability performance and comply with regulatory standards. Their study demonstrated that emission-based cost optimization provides more accurate and socially responsible decision frameworks compared to conventional economic order models. Similarly, Ruidas et al. (2021) have explained that carbon emission cost should not be considered as an external factor but as a direct component of the operational cost structure. According to his research, integrating emission-associated cost encourages industries to adopt modern green production technologies and optimize order quantities that reduce pollution without compromising profitability. Furthermore, Liao et al. (2025) have highlighted that sustainable inventory decisions not only help in environmental conservation but also enhance long-term competitiveness and brand value in global markets.

In the present study, carbon emission costs are incorporated through the parameters C_k and C_h as linear additive cost components associated with the carbon emissions in the inventory system as demonstrated by Benjaafar et al. (2013). The parameter C_k captures the environmental impact associated with each ordering and replenishment activity, such as emissions generated during order processing and transportation. Since the frequency of ordering depends on the order quantity Q, this cost component is indirectly quantity-dependent through ordering decisions. Similarly, the parameter C_h represents the environmental cost associated with holding inventory, including emissions arising from storage, warehousing and energy consumption over time. This component depends on the average inventory level and the replenishment cycle length and is therefore time-dependent in nature. Although both C_k and C_h are modelled as linear additive constants, they reflect how operational decisions influence environmental impact through inventory dynamics. Thus, the carbon cost structure adopted in this study are activity-based environmental cost approximation, which allows the environmental dimension to be incorporated into the inventory decision framework without explicitly modelling emission generation processes.

In this study, lead-time demand is modelled using an exponential distribution. The Exponential lead-time demand distribution is particularly appropriate in environments characterized by memoryless demand behavior, such as service systems, spare-parts inventory, or low-to-moderate volume settings where demand arrivals occur randomly and independently. It yields closed for expressions for expected shortages and cost components facilitating explicit analytic optimization. Johnston and Harrison (1986) examine the distributional properties of lead time demand and provide variance formulae under simple stochastic smoothing approaches, highlighting how exponential or related memoryless processes can be used to approximate random lead-time demand behavior in inventory systems.

In the proposed model, interest charged and earnings are applied only to monetary cash flows and not to environmental cost components. This modelling choice follows established trade credit and inventory financing literature, where interest is linked to capital tied up in inventory purchases and sales revenues rather than to operating or compliance-related expenses (Goyal, 1985; Chen et al., 2017). Environmental costs, including those associated with emissions or sustainability considerations, are typically treated as operating expenditures and are not subject to financing or interest accrual in practice. The model assumes that sales revenue is continuously deposited and earns interest at rate I_e. This assumption is commonly used in trade credit inventory models to represent business environments characterized by frequent sales transactions, rapid settlement and efficient financial systems (Goyal, 1985; Teng, 2002).

The present study contributes to the inventory literature by extending existing models to jointly examine the effects of trade credit financing, probabilistic lead time demand and carbon emission costs within a unified framework. The proposed framework retains analytical tractability and provides managerial insights into the interaction between financial incentives and environmental considerations in inventory decision-making. The overview of the study can be summarized as Section 2 provides the relevant literature associated with the study, Section 3 elaborates the model formulation with notations, assumptions and derivation of cost components of the total cost, Section 4 provides the algorithm to obtain the optimum solutions for the order quantity, reorder point, Section 5, 6 and 7 helps to understand the nature of the model with the help of numerical example, graphical nature of cost function and sensitivity analysis by varying the parameter values. In the final analysis, Section 8, managerial insights assess the impact of parameter variations on model outputs and Section 9 concludes the study.

2. Literature Review

In addition to environmental aspects, financial arrangements in modern commercial transactions also influence inventory decisions. One such widely used arrangement is trade credit, under which suppliers permit the purchasing firm to delay payment for a specified period without immediate financial burden. During this credit period, the buyer can sell the goods and generate revenue before paying the supplier. This facility provides the buyer flexibility in cash management, improves liquidity, enhances purchasing capacity and helps businesses maintain continuous operations even when capital availability is restricted. Trade credit is especially valuable for small and medium-sized enterprises that struggle to access traditional banking loans. When trade credit is considered in inventory modeling, the optimal order quantity often increases because buyers seek to maximize the advantage of deferred payment. This shift affects holding cost, interest earned, interest charged and total system cost making trade credit an important decision variable within modern inventory analysis. In the context of sustainability, trade credit becomes a strategic tool that provides financial flexibility for adopting green practices and carbon-reducing investments. According to Qin (2017), trade credit provides financial flexibility that supports companies in adopting emission-reduction strategies without immediate monetary burden. Qin (2017) has stated that trade credit, when combined with sustainability considerations such as carbon emission costs, offers a win-win situation for both suppliers and buyers, as it encourages higher sales, supports operational expansion and motivates buyers to invest in environmentally responsible decisions. His research highlighted that trade credit financing promotes economic growth while allowing companies to simultaneously adopt green initiatives. Sustainable inventory models that combine carbon emissions control with trade credit policies have drawn increasing attention in operations research, because they reflect the dual pressure of environmental regulation and financial incentives. Chen et al. (2017) developed a retailer's Economic Order Quantity (EOQ) model under a cap and trade system, showing that trade credit shortens ordering cycles for high emission items and influences carbon reduction strategies through financing costs.

Goyal (1985) laid foundational work showing that permissible delay in payment reduces effective financing cost for the buyer, thereby increasing order size and reducing total cost under certain conditions. Building on this, Qin (2017) examines a sustainable inventory model where a retailer receives trade credit and invests in emission-reduction technology under carbon-pricing regimes; her results highlight that favorable credit terms enable larger orders and greener investment, but the net benefit strongly depends on how emission cost is structured and regulated. Taleizadeh et al. (2022) formulate a sustainable inventory system that explicitly incorporates partial trade credit, partial backordering, price-sensitive demand and carbon emission cost. Their work is notable for combining realistic retail behaviors with environmental cost and credit terms. They derive the retailer's optimal policy and perform sensitivity analysis to reveal how partial credit and emission cost jointly influence order size, price and backorder decisions. Practically, this paper shows that adding even limited credit flexibility affects both profitability and emission outcomes. Dua et al. (2024) formulates a deteriorating-item inventory model that explicitly includes partial trade credit as the financing mechanism and integrates an emission cost term. The author provides an algorithmic solution and numerical experiments that show how partial trade credit reduces financing cost and can encourage larger order sizes, but also how emission pricing moderates this tendency. The influence of trade credit terms on inventory decisions for deteriorating products was first explored by Aggarwal and Jaggi (1995). Their model incorporated a fixed deterioration rate and allowed buyers a single credit period before payment was due. Following their work, researchers have produced a wide range of studies exploring how deferred payment options affect inventory planning for items that lose value over time. A noteworthy advancement was made by Teng (2009), who introduced an inventory model aimed at customers with higher credit risk. In his approach, buyers are required to make an initial partial payment at the time of ordering, ensuring coverage of potential default risk during the permissible credit period. Tinani and Kandpal (2017) examine how inflation and trade-credit influence optimal inventory decisions in a stochastic environment with supply uncertainty. Their model assumes non-deteriorating items sourced from suppliers who may be randomly unavailable and the buyer receives a delay in payment advantage. They find that under inflationary pressures, the benefit of trade credit increases the incentive to order larger quantities, but supply uncertainty and payment delay risk counteract this benefit, leading to modified optimal reorder points and cycle lengths.

Christata and Daryanto (2020) conducted a review of EOQ models with carbon-emission considerations and found that many early studies assume deterministic demand and ignore lead-time variability. Their review identifies that most models address storage and transportation emissions but lack integration with other complex factors. Similarly, Md Mashud et al. (2021) propose a sustainable inventory model with controllable carbon emissions for a two-warehouse system and demonstrate how green technology investment can reduce emissions and cost. In their model, carbon emission cost is explicitly incorporated into the objective function alongside traditional costs. More recently, Suvetha et al. (2025) presents a production-inventory model where carbon emission cost is tied to both production and holding-stock levels, emphasizing that emission cost should be treated as a continuous decision variable rather than a fixed penalty. Alamri et al. (2022) extend EOQ modelling by adding carbon emission cost together with inflation and inspection considerations. Their findings indicate that when emission costs are combined with inflation effects and quality inspection rules, the cycle time and order quantity change notably compared to classic EOQ and sensitivity analysis helps managers understand which parameters most influence both profit and emissions. In a study by Banu and Mondal (2016) the demand function is tied to the credit period extended by suppliers. Although this paper does not focus explicitly on emissions, it is relevant because it evidences how trade credit affects ordering decisions an effect which becomes even stronger once emission cost is added into the model. Pramanik and Maiti (2020) developed an inventory model with imprecise demand under two-level partial trade credit and showed that credit terms can motivate the buyer to order more, thereby increasing holding cost but potentially improving cash-flow and profitability.

Another notable work is Carbon Emission Sensitive Deteriorating Inventory Model by De et al. (2021), which considers production/manufacturing systems in which the emission cost is tied to inventory control decisions under deterioration and trade credit constraints. Their results highlight that when emission related costs increase, the system may shift toward smaller orders and more frequent replenishments to reduce holding-emission trade-offs. De-la-Cruz-Márquez et al. (2023) pointed out that the consideration of carbon emissions in inventory models leads to more realistic and socially responsible decision frameworks, particularly when dealing with industries facing uncertain demand and imperfect production conditions. Their research proved that incorporating carbon related costs enhances competitiveness, improves sustainability performance and aligns operations with government regulations such as carbon tax and emissions trading systems.

In addition to financial and environmental factors, uncertainty in lead time and demand is an important issue in real-world supply chains. Lead time refers to the time interval between placing an order and receiving the goods. In practice, lead time may vary due to production delays, transportation breakdowns, weather conditions, customs processing, labor shortage and unexpected disruptions such as pandemics or strikes. When lead time is uncertain, determining the right inventory level becomes more difficult. Demand that occurs during the lead time is crucial, because if inventory is insufficient, shortages may occur, leading to loss of sales, reduced customer satisfaction and damaged relationships. To model such uncertainty realistically, many researchers consider stochastic lead time demand. In a review conducted by Tinani and Sarangale (2024) emphasized that the normal distribution is the preferred distribution in various literatures despite having its limitations. Therefore, other probability distributions need to be explored for sustainable inventory management. Among various probabilistic distributions, the exponential distribution is particularly useful in modeling random demand arrivals because of its analytical simplicity and ability to describe unpredictable and memory less behavior. Modeling exponential lead time demand allows inventory planners to estimate expected shortages, safety stock and replenishment parameters more accurately. Yang and Lin (2019) examines stochastic lead time with order crossover and include an exponential lead-time case study to illustrate the method. They emphasize that exponential lead time is a reasonable assumption in many contexts and show how mis specifying the lead time distribution can materially affect inventory policy performance. The paper is therefore helpful when validating models that assume exponential lead time against more general stochastic frameworks. Indhumathy and Jayashree (2016) developed a stochastic inventory model in which lead-time demand follows an exponential distribution and price-breaks are present; they demonstrated how exponential lead-time demand affects reorder points and lot sizes compared to deterministic lead-time assumptions. Further, empirical studies on lead-time demand have shown that such demand is frequently positively skewed and difficult to approximate accurately using symmetric distributions such as the normal distribution. In particular, Bagchi et al. (1986) demonstrate that when demand is independent the exponential distribution is appropriately employed when demand during lead times are highly uncertain and exhibit substantial right-skewness. In such settings, lead time demand variability dominates the overall uncertainty making symmetric approximations inadequate. The exponential distribution, as a limiting case of the gamma family, captures situations in which delays have no characteristic scale and the likelihood of additional delay does not diminish with elapsed time. When combined with approximately normal demand per unit time, this assumption leads to a strongly right-skewed distribution of demand during lead time, for which normal approximations significantly understate stockout risk, particularly at high service levels. Consequently, the exponential distribution is best viewed as a conservative but analytically tractable representation of extreme lead time demand variability, suitable for environments characterized by long, unpredictable replenishment delays and a managerial emphasis on protection against rare but severe shortages. As described by Saidane et al. (2013) exponential lead time demand provides a reasonable approximation in inventory systems characterized by highly random and independent demand arrivals with limited temporal dependence, such as spare-parts inventories, service and maintenance operations, and emergency supply systems. Consequently, memoryless or baseline stochastic representations are commonly adopted in analytically tractable inventory models.

This study aims to understand the effect of incorporation of trade credit and carbon emission costs on the inventory system when lead time demand follows the exponential distribution. The objective is to find the optimum order quantity and reorder level and its corresponding total cost and understand the behavior of changing parameters on the overall inventory system to suggest good managerial practices which lowers the total cost while considering the financial and environmental factors in mind.

3. Model Formulation

3.1. Notations

Table 1.: Notation Used in the Mathematical Model

Notation	Description
Q	Order Quantity
r	Reorder Level
D	Average Annual Demand
K	Cost of Ordering
C_k	Carbon emission cost while ordering
h	Cost of holding an inventory
C_h	Carbon emission cost while holding
π	Backordering cost
$f(x)$	Probability distribution function of lead time demand
λ	Parameter of lead time demand distribution
T	Cycle Time
c	Unit Purchasing cost
p	Unit selling price

3.2. Assumptions

(a) It is a single item inventory system.
(b) It is a continuous review inventory system. Inventory system allows backordering.
(c) Cost of ordering, holding and backordering are known and constant.
(d) Holding cost is the carrying cost of unit per unit time.
(e) Ordering cost is the cost of per order.
(f) Backordering cost is the cost of per unit stocking out.
(g) Supplier allows a delay M, if the retailer pays by time M, no interest will be charged. After M time retailer pays interest at rate I_c on the unpaid inventory value.
(h) Retailer sells continuously at rate D, as sales revenue comes in, it can be deposited at rate I_e until it is used to pay the supplier.
(i) Cycle Time T is ratio of order quantity to average annual demand.
(j) Lead time demand has an exponential distribution with pdf $f(x) = \lambda e^{-\lambda x}$ such that $x > 0$ with average demand $\frac{1}{\lambda}$.
(k) Q is much larger than expected lead time demand.

Incorporating trade credit results in two distinct regimes. When the replenishment cycle length T does not exceed the credit period M (Case A: $T \leq M$), all inventory operations occur while credit is still available. Consequently, the lead-time variability continues to govern the holding and backordering components exactly as in the baseline model; only the financing terms are altered to reflect interest earned or saved under credit. In contrast, when the cycle is longer than the credit period (Case B: $T > M$), a portion of the cycle must be financed after the credit expires. Although the inventory path and stochastic lead-time behavior remain unchanged, the interest cost structure adjusts to reflect the mixed credit and post-credit intervals. To incorporate trade credit into the model, we introduce a financing term FC(Q) which represents the net finance cost per unit time. The total expected cost becomes

$$TC(Q,r) = TC_0(Q,r) + FC(Q) \tag{1}$$

where $TC_0(Q,r)$ denotes the baseline operating cost (holding, ordering, backordering and carbon-related components),

$$TC_0(Q,r) = \text{Ordering cost} + \text{Holding Cost} + \text{Backordering cost} \tag{2}$$

$$TC_0(Q,r) = \frac{D}{Q}(K + C_k) + (h + C_h)\left(\frac{Q}{2} + r - \frac{1}{\lambda}\right) + \frac{\pi D}{Q}\frac{e^{-\lambda r}}{\lambda}. \tag{3}$$

Whereas $(\frac{Q}{2} + r - \frac{1}{\lambda})$ is the on-hand inventory level and $\frac{e^{-\lambda r}}{\lambda}$ are the number of backorders when lead time demand follows the exponential distribution and $FC(Q)$ captures the net effect of interest earned during the credit period and interest paid after the credit period ends. Interest is applied only to monetary values (inventory value and sales revenue), not to environmental cost parameters such as C_h, C_k, etc.

Case A: Short Cycle ($\mathbf{T = Q/D \leq M}$)

When the cycle length does not exceed the credit period, the buyer never exits the free-credit window consequently, no interest is paid. However, the buyer earns interest on sales revenue collected during the cycle, as each sale occurs before the credit period ends.
At time $t \in [0, T]$, revenue amount generates and this amount accumulates interest for the remaining duration $(M - t)$. The total interest earned per cycle is

$$IE_{T\leq M}(Q) = I_e \int_0^T pD(M - t)\, dt = I_e pD \left(MT - \frac{T^2}{2} \right). \tag{4}$$

Since this is a benefit, the net finance cost per unit time is

$$FC_{T\leq M}(Q) = -\frac{IE_{T\leq M}(Q)}{T} = -I_e pD \left(M - \frac{T}{2} \right). \tag{5}$$

Substituting $T = \dfrac{Q}{D}$ yields

$$FC_{T\leq M}(Q) = -I_e pDM + \frac{I_e p}{2} Q. \tag{6}$$

This relationship shows that more frequent replenishments (smaller Q) increase the frequency of credit benefits, while larger Q increases the financing cost.

Case B: Long Cycle ($T = Q/D > M$)
When the cycle extends beyond the credit period, the buyer earns interest (on sales within the credit window) and pays interest (on inventory held after the credit expires). The net cost consists of these two components.

Interest earned on revenue for T ≤ M
Sales during generate revenue that earns interest until time . The interest earned per cycle is

$$IE_{T>M}(Q) = I_e \int_0^M pD(M - t)\, dt = \frac{I_e pDM^2}{2}. \tag{7}$$

Thus, the earned interest per unit time is

$$\frac{IE_{T>M}(Q)}{T} = \frac{I_e pDM^2}{2T}. \tag{8}$$

Interest charged on inventory for $t \in [M, T]$
During the interval $[M, T]$, the inventory level follows the standard EOQ depletion pattern

$$I(t) = Q - Dt.$$

Only the inventory held after the credit period is subject to interest charges. Therefore, the interest paid per cycle is

$$IC_{T>M}(Q) = I_c c \int_M^T (Q - Dt)\, dt = \frac{I_c c D (T-M)^2}{2}, \tag{9}$$

where we have used the relation $Q = DT$. Hence, the charged interest per unit time is

$$\frac{IC_{T>M}(Q)}{T} = \frac{I_c c D (T-M)^2}{2T}. \tag{10}$$

Net finance cost for the long-cycle case
Combining the components gives

$$FC_{T>M}(Q) = \frac{I_c c D (T-M)^2}{2T} - \frac{I_e p D M^2}{2T} \tag{11}$$

Substituting $T = \frac{Q}{D}$ yields the standard linear-plus-inverse EOQ structure:

$$FC_{T>M}(Q) = \frac{I_c c}{2} Q - I_c c M D + \frac{M^2 D^2}{2Q}(I_c c - I_e p). \tag{12}$$

This expression is consistent with classical EOQ models under trade credit, featuring a linear term in Q, a constant term, and a reciprocal term $1/Q$.

The original expected cost per unit time, excluding financial effects, is $TC_0\,(Q, r)$ which is unaffected by the introduction of trade credit, since interest is applied only to monetary values and not to physical holding-cost terms.
Collecting all components, the total expected cost is

$$TC(Q,r) = TC_0(Q,r) + \begin{cases} FC_{T\le M}(Q), & Q \le DM, \\ FC_{T>M}(Q), & Q > DM. \end{cases} \tag{13}$$

This function preserves the structure of the classical (Q,r) model while incorporating trade-credit financing effects through a simple, analytically tractable piecewise term. The resulting optimization problem remains well-behaved, enabling closed-form expressions for first-order conditions and efficient numerical solution.
Further, the optimal reorder level r^* and optimal order quantity Q^* can be derived when the trade-credit financing term $FC(Q)$ is incorporated into the total cost function and it follows that,

$$\frac{\partial FC(Q)}{\partial r} = 0 \tag{14}$$

Hence, the first-order condition for the reorder point is identical to that of the original model. Differentiating the total cost with respect to r yields

$$\frac{\partial TC}{\partial r} = (h + C_h) - \frac{\pi D}{Q} e^{-\lambda r} = 0 \tag{15}$$

Solving for r gives the closed-form expression

$$r^* = -\frac{1}{\lambda} \ln\left(\frac{(h + C_h)Q}{\pi D} \right) \tag{16}$$

Thus, the introduction of trade credit does not alter the economic interpretation of the reorder point: it continues to balance the trade-off between under-stocking and over-stocking. As a result, the optimal reorder level depends only on operational and demand-related parameters, not on financing terms. The derivative of the baseline cost component is

$$\frac{\partial TC_0}{\partial Q} = -\frac{D(K + C_k)}{Q^2} + \frac{h + C_h}{2} - \frac{\pi D e^{-\lambda r}}{\lambda Q^2} \tag{17}$$

The first-order condition for Q becomes

$$\frac{\partial TC}{\partial Q} = \frac{\partial TC_0}{\partial Q} + \frac{dFC(Q)}{dQ} = 0 \tag{18}$$

with the derivative of $FC(Q)$ determined by whether Q lies within the short-cycle or long-cycle regime.

Case A: $T \leq M$ — Cycle within the credit period ($Q \leq DM$)

The finance cost in this region is

$$FC_{T \leq M}(Q) = -I_e p DM + \frac{I_e p}{2} Q \tag{19}$$

the derivative is simply

$$FC'_{T \leq M}(Q) = \frac{I_e p}{2} \tag{20}$$

Substituting into the first-order condition gives

$$0 = -\frac{D(K + C_k)}{Q^2} + \frac{h + C_h}{2} - \frac{\pi D e^{-\lambda r}}{\lambda Q^2} + \frac{I_e p}{2} \tag{21}$$

Rearranging,

$$\frac{h + C_h + I_e p}{2} = \frac{D}{Q^2} \left(K + C_k + \frac{\pi e^{-\lambda r}}{\lambda} \right) \tag{22}$$

Solving for Q yields a closed-form EOQ-type solution

$$Q^*_{T \leq M} = \sqrt{\frac{2D\left(K + C_k + \frac{\pi e^{-\lambda r}}{\lambda}\right)}{h + C_h + I_e p}} \tag{23}$$

The term $I_e p$ enters the denominator, increasing the effective holding-cost rate. Because earned interest offsets part of the holding cost, the optimal order quantity decreases. This result is fully consistent with the classical finding that trade credit encourages more frequent replenishment.

Case B: T > M - Cycle exceeds credit period (Q > DM)

For the long-cycle case, the finance term is

$$FC_{T>M}(Q) = \frac{I_c c}{2}Q - I_c cMD + \frac{M^2 D^2}{2Q}(I_c c - I_e p) \tag{24}$$

Differentiating,

$$FC'_{T>M}(Q) = \frac{I_c c}{2} - \frac{M^2 D^2}{2Q^2}(I_c c - I_e p) \tag{25}$$

The first-order condition becomes

$$0 = -\frac{D(K + C_k)}{Q^2} + \frac{h + C_h}{2} - \frac{\pi D e^{-\lambda r}}{\lambda Q^2} + \frac{I_c c}{2} - \frac{M^2 D^2}{2Q^2}(I_c c - I_e p) \tag{26}$$

Grouping the terms in $1/Q^2$

$$\frac{h + C_h + I_c c}{2} = \frac{1}{Q^2}\left[D\left(K + C_k + \frac{\pi e^{-\lambda r}}{\lambda}\right) + \frac{M^2 D^2}{2}(I_c c - I_e p)\right] \tag{27}$$

Solving for gives

$$Q^*_{T>M} = \sqrt{\frac{2D\left(K + C_k + \frac{\pi e^{-\lambda r}}{\lambda}\right) + M^2 D^2 (I_c c - I_e p)}{h + C_h + I_c c}} \tag{28}$$

In this region, both interest paid (after the credit period) and interest earned (during the credit period) influence the optimal order quantity. The numerator includes a term proportional to M^2, representing the accumulated inventory-financing burden during the post-credit part of the cycle. The denominator increases by $I_c c$, reflecting the effective capital-cost component of inventory. As a result, $Q^*_{T>M}$ may be larger or smaller than in the base model depending on the relative magnitudes of I_e and I_c.

4. Algorithm to obtain the optimal solution

The coupled first-order conditions for Q and r generally do not yield a simultaneous closed-form solution when trade credit is included. However, because each variable admits a closed-form update conditional on the other, a simple and numerically stable iterative scheme can be used. This approach parallels the fixed-point algorithm employed earlier for the non-credit model, with the addition of a regime check to account for the piecewise financing term. Further iterative procedure can be adopted to obtain the

optimal values.
Step 1: Set

$$Q_1 = \sqrt{\frac{2D(K + C_k)}{h + C_h}},$$

which is the value obtained by ignoring backorders and trade credit.

Step 2: Find r_n satisfying

$$r_n = -\frac{1}{\lambda} \ln \left(\frac{(h + C_h)Q_n}{\pi D} \right).$$

Step 3: Determine whether the current iterate Q_n lies in the short-cycle or long-cycle region:

- If $Q_n \leq DM$ (cycle within credit period), use the Case A.
- If $Q_n > DM$ (cycle exceeds credit period), use the Case B.

Step 4: Depending on the region identified in Step 3

Case A: $Q_n \leq DM$, use the closed-form solution for the short-cycle region:

$$Q_{n+1} = \sqrt{\frac{2D\left(K + C_k + \frac{\pi e^{-\lambda r_n}}{\lambda}\right)}{h + C_h + I_e p}} \tag{29}$$

Case B: $Q_n > DM$, use the closed-form solution for the long-cycle region:

$$Q_{n+1} = \sqrt{\frac{2D\left(K + C_k + \frac{\pi e^{-\lambda r_n}}{\lambda}\right) + M^2 D^2 (I_c c - I_e p)}{h + C_h + I_c c}} \tag{30}$$

Step 5: If $\mid Q_{n+1} - Q_n \mid \leq \varepsilon$, for a prescribed tolerance (e.g., $\varepsilon = 10^{-6}$), then terminate the procedure and report the optimum values otherwise return to step 2.

The stopping criterion is based on the change in successive values of the order quantity. Since the reorder level is explicitly determined by the order quantity, convergence of Q implies convergence of both decision variables. Joint convexity of the total cost function ensures that once successive updates differ by less than a small tolerance, the solution is sufficiently close to the global optimum. Numerical results provided below confirm that this criterion provides stable and efficient convergence.

5. Numerical Example

Example 1: To illustrate the application of the proposed (Q, r) model with a continuous review policy, where lead time demand follows the exponential distribution with mean $1/\lambda$ and trade credit, considering the parameter values annual demand: D=1000 units, Ordering cost: $K = 50$ *per order*, Ordering carbon cost: $C_k = 5$ *per order*, Physical holding cost: $h = 3$ *per unit*, Carbon holding cost: $C_h = 1$ *per unit*, Backordering cost: $\pi = 20$ *per unit item*, Mean lead time: $1/\lambda = 10$ units (hence $\lambda = 0.1$), Credit period: $M = 0.1$ years, Unit purchasing cost: c=10, Unit selling price: p=15, Interest charged on inventory: $I_c = 0.10$, Interest earned on sales revenue: $I_e = 0.05$

Above numerical problem yields the optimal values with optimal order quantity $Q^* = 158.21$ units, optimal reorder level $r^* = 34.53$ units with minimum total cost $TC^* = 789.196$.

Example 2: Under stricter environmental considerations such as increased emphasis on emission during transportation and storage high values of carbon of emission costs where $C_k = 15$ *per order* and $C_h = 3$ *per unit* and keeping all the other parameter values as above optimum order quantity is $Q^* = 146.4193$ units, optimal reorder level $r^* = 31.25254$ units with minimum total cost $TC^* = 1052.45$.

Example 3: To understand the greater financial flexibility under the common competitive markets another example can explored where $M = 0.5$ years while other parameter values are kept unchanged, which results into optimum order quantity as $Q^* = 160.8311$ units, optimal reorder level $r^* = 34.36839$ units with minimum total cost $TC^* = 486.4211$.

The results indicate that increased emission related costs lead to higher total costs and lower optimal order quantities, reflecting the impact of environmental considerations on inventory decisions. At the same time, the longer trade credit period provides partial financial relief, mitigating the increase in total cost. Importantly, the structural properties of the model and the convergence behaviour of the solution algorithm remain unchanged, confirming the robustness of the proposed framework.

6. Non-linearity of the cost function

The convexity of the cost function and the convergence of the proposed algorithm are established by examining the Hessian matrix in each region. The first-order partial derivative for baseline cost function with respect to r is

$$\frac{\partial TC}{\partial r} = (h + C_h) - \frac{\pi D}{Q} e^{-\lambda r} \tag{31}$$

The second-order derivative with respect to the reorder level is

$$\frac{\partial^2 TC}{\partial r^2} = \lambda \frac{\pi D}{Q} e^{-\lambda r} > 0 \tag{32}$$

which shows that the baseline cost function is strictly convex in r for any feasible Q. The cross-partial derivatives are

$$\frac{\partial^2 TC}{\partial Q \partial r} = \frac{\pi D}{Q^2} e^{-\lambda r}, \frac{\partial^2 TC}{\partial r \partial Q} = \frac{\pi D}{Q^2} e^{-\lambda r} \tag{33}$$

Case A: T = Q/D ≤ M (short cycle within the credit period) In this region, the finance cost is

$$FC_{T\leq M}(Q) = -I_e pDM + \frac{I_e p}{2} Q \tag{34}$$

which is linear in Q. Hence,

$$\frac{\partial^2 FC_{T\leq M}(Q)}{\partial Q^2} = 0. \tag{35}$$

The second-order partial derivative of the total cost with respect to Q is therefore

$$\frac{\partial^2 TC}{\partial Q^2} = \frac{2D(K+C_k)}{Q^3} + \frac{2\pi D}{\lambda Q^3} e^{-\lambda r} \tag{36}$$

which is strictly positive for all $Q > 0$. The Hessian matrix of $TC(Q, r)$ in Case A is given by

$$H_A(Q,r) = \begin{pmatrix} \frac{2D(K+C_k)}{Q^3} + \frac{2\pi D}{\lambda Q^3} e^{-\lambda r} & \frac{\pi D}{Q^2} e^{-\lambda r} \\ \frac{\pi D}{Q^2} e^{-\lambda r} & \lambda \frac{\pi D}{Q} e^{-\lambda r} \end{pmatrix} \tag{37}$$

Since $\frac{\partial^2 TC}{\partial r^2} > 0$ and

$$\det(H_A) = \left(\frac{2D(K+C_k)}{Q^3} + \frac{2\pi D}{\lambda Q^3} e^{-\lambda r}\right)\left(\lambda \frac{\pi D}{Q} e^{-\lambda r}\right) - \left(\frac{\pi D}{Q^2} e^{-\lambda r}\right)^2 > 0. \tag{38}$$

the Hessian matrix $H_A(Q, r)$ is positive definite. Hence, the total cost function is jointly convex in (Q, r) for $Q \leq DM$.

Case B: T = Q/D > M (long cycle beyond the credit period)
In this region, the finance cost is

$$FC_{T>M}(Q) = \frac{I_c c}{2} Q - I_c cMD + \frac{M^2 D^2}{2Q}(I_c c - I_e p) \tag{39}$$

The second-order derivative of the finance cost with respect to Q is

$$\frac{\partial^2 FC_{T>M}(Q)}{\partial Q^2} = \frac{M^2 D^2}{Q^3}(I_c c - I_e p) \tag{40}$$

Thus, the second-order derivative of the total cost with respect to Q becomes

$$\frac{\partial^2 TC}{\partial Q^2} = \frac{2D(K+C_k)}{Q^3} + \frac{2\pi D}{\lambda Q^3} e^{-\lambda r} + \frac{M^2 D^2}{Q^3}(I_c c - I_e p) \tag{41}$$

the sum of all terms remains positive due to the strictly positive ordering and shortage components. The Hessian matrix in Case B is

$$H_B(Q,r) = \begin{pmatrix} \frac{2D(K+C_k)}{Q^3} + \frac{2\pi D}{\lambda Q^3}e^{-\lambda r} + \frac{M^2D^2}{Q^3}(I_c c - I_e p) & \frac{\pi D}{Q^2}e^{-\lambda r} \\ \frac{\pi D}{Q^2}e^{-\lambda r} & \lambda \frac{\pi D}{Q}e^{-\lambda r} \end{pmatrix} \tag{42}$$

The determinant of $H_B(Q,r)$ satisfies

$$\det(H_B) > 0,$$

and the leading principal minors are positive. Therefore, the Hessian matrix is positive definite, and the total cost function is jointly convex in (Q,r) for $Q > DM$.
Since the total cost function is jointly convex in (Q,r) in both regions, the stationary point obtained from the first-order conditions corresponds to a unique global minimum. Numerical examples also show that the algorithm converges smoothly and requires only a small number of iterations for a wide range of parameter values.

To examine the structural behaviour of the total cost function $TC(Q,r)$, the three-dimensional surface plots can be generated shown in Figure 1. These diagrams depict the cost surface as a function of both the order quantity Q and the reorder level r, using the parameter values specified in the numerical example. The plots provide a visual verification of the model's convexity properties and highlight the unique optimal solution identified by the iterative algorithm. The surfaces exhibit a smooth, curved geometry across the admissible ranges of Q and r, rising sharply as either decision variable deviates from its optimal neighborhood. The steep increase observed for small values of Q correspond to excessive ordering costs, backordering cost and the influence of the $1/Q^2$ terms in the cost function. Conversely, very large values of Q generate elevated holding and financing costs, producing the upward curvature on the opposite side of the surface. Similarly, when the reorder point r becomes too small, expected backorders dominate, driving the cost sharply upward. When r becomes excessively large, the holding-cost term grows, again elevating total cost. The interplay of these opposing forces results in a surface that is convex in the joint space of Q and r, supporting the existence and uniqueness of a global minimum. This convex structure is fully consistent with the analytical form of the objective function, which combines linear, quadratic and rational terms in a manner known to yield unimodal behaviour under standard EOQ-type conditions. The graphical evidence therefore reinforces the theoretical expectation that the proposed model possesses a well-defined optimum. Both surface plots display a red marker indicating the numerically computed optimal inventory policy and is located at the global minimum of the surface. The red dot lies precisely at the lowest region of the cost landscape, highlighting the convergence of the iterative (Q,r) procedure used. The position and shape of the contours surrounding the red point demonstrate that the curvature is relatively steep in the directions of both variables, implying a well-conditioned minimization problem and further supporting the numerical stability of the proposed solution procedure. The visualization also reveals that small deviations from Q^* or r^* lead to noticeable increases in cost, underscoring the managerial importance of selecting appropriate replenishment decisions.

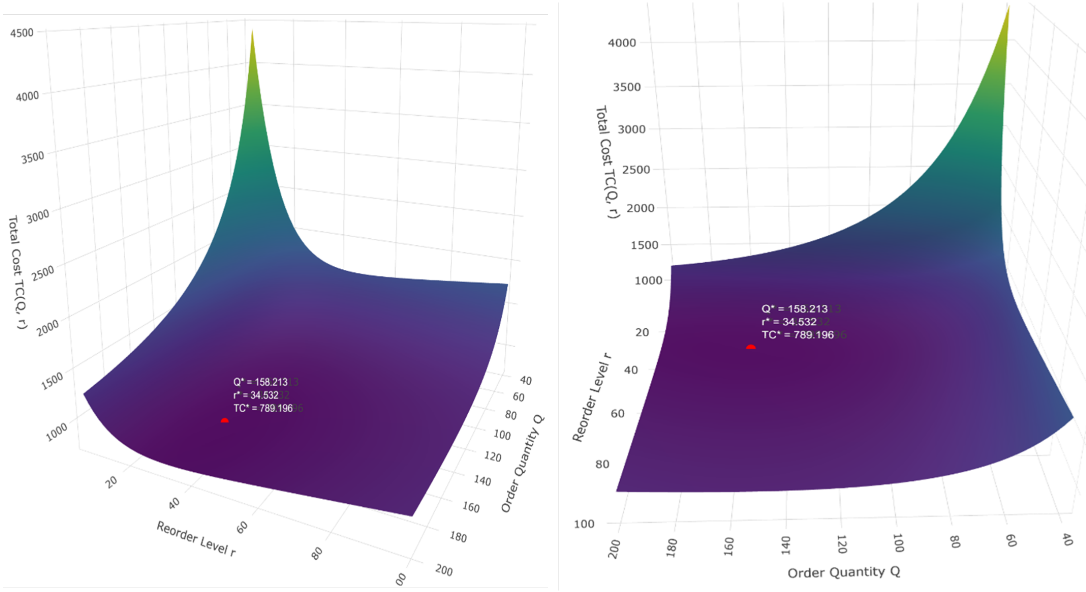

Figure 1.: Non-linearity of the Cost Function for Numerical Example 1 where Case B applies

7. Sensitivity Analysis and Discussions

To examine how changes in key parameters influence the optimal order quantity, reorder level and total cost, a sensitivity analysis is performed. The analysis considers a range of values for $1/\lambda$ while holding all other parameters constant. Additionally, the effects of varying one cost related parameter at a time namely, the holding cost, ordering cost and carbon emission costs associated with ordering and holding, backordering cost, trade credit period, interest charged and interest earned are evaluated while maintaining $1/\lambda = 0.10$ for all subsequent computations. The resulting outcomes are summarized in Table 2. The sensitivity analysis provides valuable insights into how key operational, financial, and environmental parameters influence the optimal inventory policy. By varying one parameter at a time while holding all others constant, several important behavioural patterns emerge. Increasing the mean lead time demand leads to higher reorder levels r^* as firms must hold more safety stock to buffer against greater uncertainty reflecting in a higher total cost. The optimal order quantity Q^* increases steadily as the carbon emission cost during ordering rises as higher ordering costs incentivize placing fewer but larger orders. The reorder point r^* remains nearly unchanged, reflecting the fact that r^* depends more strongly on lead time demand uncertainty than on ordering cost. Higher emission cost during ordering translates directly into gradual increase in total costs. As the carbon emission cost during holding increases, both Q^* and r^* decrease. Larger holding emission cost penalizes inventory accumulation, motivating smaller and more frequent replenishments with very small decrease in the reorder level. Total cost increases sharply with higher holding emission cost, emphasizing the need for managers to minimize storage and carbon related holding expenses through efficient facility operations or greener storage technologies. An increase in backordering cost leaves Q^* unchanged but raises the reorder level r^*. Because backorders become more costly, the firm compensates by increasing safety stock. Improving service levels and reducing stockouts through better forecasting or

Table 2.: Sensitivity Analysis for Varying Parameter Values

Parameter	Parameter Values	Q^*	r^*	Total Cost
λ	0.05	166.8509	68.0018	926.2620
	0.10	158.2132	34.5325	789.1959
	0.15	155.4281	23.1401	743.0342
	0.20	154.0533	17.3995	719.8645
	0.25	153.2341	13.9409	705.9343
C_k	3.50	156.2026	34.6604	779.6544
	4.00	156.8758	34.6174	782.8485
	4.50	157.5460	34.5748	786.0289
	5.00	158.2132	34.5325	789.1959
	5.50	158.8774	34.4906	792.3496
	6.00	159.5388	34.4490	795.4901
	6.50	160.1972	34.4079	798.6177
C_h	0.25	170.5245	35.8595	708.7727
	0.50	166.0828	35.3824	736.2111
	0.75	161.9939	34.9417	763.0023
	1.00	158.2132	34.5325	789.1959
	1.25	154.7039	34.1506	814.8354
	1.50	151.4350	33.7925	839.9592
	1.75	148.3803	33.4556	864.6011
π	5	158.2132	20.6696	733.7441
	10	158.2132	27.6010	761.4700
	15	158.2132	31.6557	777.6886
	20	158.2132	34.5325	789.1959
	25	158.2132	36.7639	798.1216
	30	158.2132	38.5872	805.4145
	35	158.2132	40.1287	811.5805
M	0.025	156.6447	34.6321	856.7521
	0.050	156.9597	34.6120	833.2468
	0.075	157.4833	34.5787	810.7313
	0.100	158.2132	34.5325	789.1959
	0.125	159.1465	34.4737	768.6270
	0.150	160.2793	34.4028	749.0077
I_c	0.08	160.2882	34.4022	786.9921
	0.09	159.2319	34.4683	788.1097
	0.10	158.2132	34.5325	789.1959
	0.11	157.2299	34.5948	790.2520
	0.12	156.2803	34.6554	791.2794
I_e	0.04	159.2085	34.4698	793.9215
	0.05	158.2132	34.5325	789.1959
	0.06	157.2113	34.5960	784.4404
	0.07	156.2026	34.6604	779.6544
	0.08	155.1870	34.7256	774.8373

supplier coordination can reduce cost exposure. A longer credit period reduces total cost and increases the optimal order quantity. The cost savings arise because firms earn more interest on early sales and defer cash outflows longer. When the supplier charges higher interest the optimum order quantity reduces keeping the reorder level almost constant while total cost increases. Managers should be cautious when accepting trade-credit terms with high post-credit interest rates, as they may diminish the financial benefits of credit. Higher interest earned on sales revenue reduces total cost and encourages smaller order quantities. The firm gains financially from selling earlier and accumulating interest during the credit window making frequent replenishment cycles more attractive. This highlights the importance of efficient receivables management as organizations with stronger cash-management capabilities can capitalize more effectively on trade-credit incentives. Reducing lead time demand uncertainty emerges as one of the most effective strategies for lowering overall inventory costs by reducing safety stock requirements. In contrast, higher holding costs discourage the accumulation of excess inventory, thereby motivating firms to adopt smaller and more frequent replenishment cycles. The results further demonstrate that trade credit particularly an extended credit period M can significantly decrease total cost, underscoring the strategic importance of negotiating favourable credit terms. The financial parameters also play a crucial role as higher interest earned on sales revenue incentivizes more frequent ordering, whereas higher interest charged on inventory after the credit period encourages more conservative ordering behaviour.

8. Managerial Insights

The results offer several managerial insights for balancing operational efficiency, financial flexibility, and environmental responsibility. Carbon-related costs have a substantial impact on inventory decisions, encouraging smaller order quantities and leaner inventory policies as emission-related ordering and holding costs rise. This indicates that environmental considerations can materially reshape replenishment strategies, particularly in regulated or sustainability focused markets. Trade credit plays a mitigating role by easing financial pressure, as longer credit periods reduce effective holding costs and can partially offset higher environmental expenses. Consequently, negotiating favorable credit terms becomes an important financial lever when compliance costs are unavoidable. In contrast, environmental and financial parameters affect inventory decisions differently: emission costs mainly influence order quantities and total cost, whereas reorder levels are driven primarily by demand uncertainty and shortage penalties. Finally, the stability of the optimal policy across a wide range of parameter values suggests that the proposed model provides a reliable planning framework under varying environmental and financial conditions. Overall, the findings emphasize that sustainable inventory management requires an integrated view of environmental impact and financial policy. Managers who jointly consider emission costs and trade credit terms can achieve better cost performance. The study demonstrates that sustainability oriented inventory policies can be combined with financial efficiency, provided that environmental costs and trade credit arrangements are managed in a coordinated manner.

9. Conclusion and Future Research Direction

This study proposes an enhanced (Q, r) inventory model that jointly incorporates trade credit, carbon-emission costs, backordering costs and probabilistic lead time demand. By providing closed-form solutions under different credit regimes and a simple computational procedure, the model is readily implementable in real inventory systems. The numerical analysis confirms the stability and robustness of the optimal policy across a wide range of parameter settings, enhancing its practical usability. Overall, the framework offers a unified and operationally meaningful approach for firms facing financial and environmental constraints in inventory planning. Further, exponential distribution may not adequately represent all supply chain environments, particularly high-volume retail systems. Accordingly, extending the proposed framework to more general gamma distributions or alternative lead-time demand models constitutes a natural and valuable direction for future research. While no explicit regulatory mechanism is assumed, the proposed framework is sufficiently flexible to accommodate alternative policy-driven emission cost structures if required such as carbon tax, cap and trade system or emission quota policy.

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